Robert Lamb,

CSCI 260, Written Assignment 1

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1. Given the recurrence equation:, then

Proof (by induction):

Base Case: Let n = 1, then , thus true for n =1.

Inductive Hypothesis: Suppose that .

Inductive Step: Let n = k + 1, then

.

Thus, by induction, for all .

1. The big-Oh characterization, in terms of n, of the running time of:

There are two assignments for s to start, plus n trips through the loop, each having four elementary operations: an addition, an assignment, an increment, and a comparison. Thus there are 4(n) +2 operations to perform, which fall into the O(n) family.

There are two assignments for p to start, plus 2n trips through the loop, each have 4 more elementary operations, a multiply, an assignment, an increment, and a comparison. Thus there are elementary operations, which falls into the O(n) family.

There are two assignments to start, then there are n2 trips through the loop, each trip having four elementary operations. Thus there are 2 + 4(n2) which falls into the (n2) family.

There are three assignments to start. Then there are four elementary operations being performed per round of the j loop. For the first run there will be 1 trip, second round will have 2 trips, third has 3, leading to a total of Thus there are Which is part of the O(n2) family.

1. Given n numbers, methods for finding the smallest and largest in less than comparisons:
   1. Suppose (n is even). Then suppose there are two empty sets, for Bigger and Smaller.
      1. Exhaustively compare two random numbers at a time, placing the smaller into the set S and the larger into set B. This is comparisons, and each set now has numbers.
      2. Perform a “bubble sort” once through for each set S and B. WLOG: Compare two numbers in the set, if one is larger keep it and discard the other. Continue this for all numbers in the set. This will require comparisons because there are numbers in the set and you start the first comparison with a pair of numbers. (ie first and second numbers counts as the first compare, third number is second compare, etc.).
      3. We now have the largest and the smallest as the final numbers from each set. The total number of comparisons is : . That is compares to generate B and S, and compares for each set.
      4. Thus if n is even we need at most comparisions.
   2. Suppose (n is odd), and we have two empty sets B and S:
      1. Leave one number to the side, which we will refer to as the extra, to generate an even number of n – 1.
      2. We have shown above that the number of comparisons is for an even number, which becomes comparisons.
      3. This just leaves the extra one, which needs at most two comparisons to verify that it is not larger than the largest from B, nor smaller than the smallest from S.
      4. Thus the largest and smallest is found with at most comparisons if n is odd.
2. Characterizing recurrence equations using the master theorem:

Using

Let

* 1. : Let
     1. Then
     2. This is clearly
  2. , then